E-Math Topical Sampler — v1

EMATH / topical

Q1. (a) Expand and simplify $(2x - 3)(x + 5) - (x - 4)^2$. [3] (b) Factorise completely $6xy - 9y + 4x - 6$. [2] (c) Solve $2x^2 - 1x - 3 = 0$. [2] [7 marks]

(a)

(b)

(c)

Q2. (a) Simplify $(x^2 - 9) / (3x^2 + 12x + 9)$. [3] (b) Given that $y = (5x - 2) / (x + 3)$, make x the subject of the formula. [3] [6 marks]

(a)

(b)

Q3. (a) Express $y = x^2 - 4x - 10$ in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3] (b) Hence state the coordinates of the turning point of the curve. [1] [4 marks]

(a)

(b)

Q4. The variables x and y are connected by $y = x^3 - 4x + 3$. Some corresponding values of x and y are given in the table.

x	-2	-1	0	1	2	3
y	3	6	3	0	3	18

(a) Calculate the value of y when $x = 2$. [1]
(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 2 units on the y-axis, draw the graph of $y = x^3 - 4x + 3$ for $-2 \le x \le 3$. [3]
(c) Use your graph to find the values of x for which $x^3 - 4x + 3 = 0$. [2]
(d) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2$. [2]
(e) By drawing a suitable straight line on the same grid, solve the equation $x^3 - 4x + 3 = 3x - 1$. [3] [11 marks]

(a)

(b)

(c)

(d)

(e)

Q5. The curve `y = x² + 8x + 3` is given.

(a) Express `y = x² + 8x + 3` in the form `(x + 4)² + -13`. [2]

(b) State the coordinates of the turning point. [1]

(c) State the equation of the line of symmetry. [1] [4 marks]

(a)

(b)

(c)

Q6. The points A(-3, 1) and B(1, 5) lie on a coordinate plane. The point C(5, 8) lies above the line AB.

(a) Find the equation of the line AB in the form `y = mx + c`. [2]

(b) Find the equation of the line through C parallel to AB. [2]

(c) Find the coordinates of the intersection of the line from part (b) with the line `y = -1x + -5`. [2]

(d) Find the length of AB, leaving your answer in surd form. [2] [8 marks]

(a)

(b)

(c)

(d)

Q7. In the diagram, A, B, C, D lie on a circle centre O. TA is tangent to the circle at A. Angle ADC = 118° and angle BAT = 40°.
Find, giving a reason for each answer,
(a) angle ABC. [2] (b) angle ACB. [2] (c) reflex angle AOC. [2] [6 marks]

(a)

(b)

(c)

Q8. In the diagram, angle ACB = angle DAB, AB = 6 cm and BC = 2 cm.
(a) Show that △ABC and △DBA are similar. [2] (b) Find the length of CD. [2] (c) Given that the area of △DBA is 36 cm^2, find the area of △ABC. [2] [6 marks]

(a)

(b)

(c)

Q9. Two concentric circles have centre O. PQ is a diameter of the larger circle and RS is a diameter of the smaller circle. PS and RQ are tangents to the smaller circle. The radius of the larger circle is 17 cm and the radius of the smaller circle is 8 cm.
(a) Show that $\triangle PSO \cong \triangle QRO$, stating your reason clearly. [2] (b) Calculate the area of $\triangle QRO$. [3] [5 marks]

(a)

(b)

Q10. In a horizontal triangle ABC, AB = 20 m, BC = 30 m and angle ABC = 95°.
(a) Calculate the length of AC. [2]
(b) Calculate angle BAC. [2]
(c) Find the shortest distance from B to AC. [2]
(d) A vertical flagpole of height 12 m stands at A. A person walks along BC. Find the greatest angle of elevation of the top of the flagpole from a point on BC. [3] [9 marks]

(a)

(b)

(c)

(d)

Q11. Three points P, Q, R lie on horizontal ground. Q is due east of P with PQ = 60 m. The bearing of R from P is 45° and the bearing of R from Q is 315°.

(a) Find angle QPR, angle PQR and angle PRQ. [2]

(b) Find the distance QR. [3]

(c) Find the area of triangle PQR and the shortest distance from R to PQ. [2] [7 marks]

(a)

(b)

(c)

Q12. A sector OAB has centre O and radius 15 cm. Angle AOB = 1.2 radians. M is the midpoint of OB. A line from A to M divides the sector into two regions.

(a) Find the length of arc AB. [1]

(b) Find the area of sector OAB. [2]

(c) Find the length of AM. [2]
(d) Find the area of the smaller region (bounded by arc AB, the line AM, and part of OB). [3] [8 marks]

(a)

(b)

(c)

(d)

Q13. Mrs Tan exchanges SGD 2500 for Malaysian ringgit at `1 SGD = 3.00 MYR`. He spends MYR 3000 on accommodation and food, and buys 100 litres of fuel at MYR 2.40 per litre. He exchanges the remaining MYR back at `1 SGD = 3.10 MYR` and receives SGD 80.
(a) Calculate the MYR remaining before he converted back. [2]
(b) Show, with working, whether he has enough to cover the return trip (assume same fuel needs). [2]
(c) A bank offers the conversion at `1 SGD = 2.95 MYR` but charges a 1.5% conversion fee. Compare this against the original rate and justify which is better. [3] [7 marks]

(a)

(b)

(c)

Q14. A home appliance shop buys a air fryer at a cost price. The shopkeeper marks up the cost price by 50% and lists it at a marked price of $1200. During a sale, the air fryer is sold at a 30% discount on the marked price, and the shopkeeper still makes a profit of 5% on the cost price.
(a) Find the marked price of the air fryer. [Already given as $1200; verify by computation.] [1]
(b) Find the selling price after the discount. [1]
(c) Find the original cost price of the air fryer. [2] [4 marks]

(a)

(b)

(c)

Q15. Mdm Noor invests a principal of $10000 with OCBC at 4% per annum compounded yearly for 5 years.
(a) Write down the formula for the total amount `A` after `n` years in terms of `P` and `r`. [1]
(b) Calculate the total amount in the account after 5 years, giving your answer correct to the nearest cent. [3] [4 marks]

(a)

(b)

Q16. A survey records the lengths, x cm, of 100 fish.

Grouped frequency table (class width 10 cm; upper bounds 10, 20, 30, 40, 50, 60): $0 < x \le 10$: 20; $10 < x \le 20$: 19; $20 < x \le 30$: 13; $30 < x \le 40$: 15; $40 < x \le 50$: 15; $50 < x \le 60$: 18.

(a) Copy and complete the cumulative frequency table. [1]
(b) On the grid, draw a cumulative frequency curve. [2]
(c) Use your curve to estimate the median, the interquartile range, and the 20th percentile. [4]
(d) A fish is considered "oversized" if their length exceeds 30 cm. Find the number of oversized fish. [2]
(e) In a second survey of 100 fish at a different site, the values had the same median but a larger interquartile range. Describe how the cumulative frequency curve for this second survey would differ from the curve drawn in (b). [2] [11 marks]

(a)

(b)

(c)

(d)

(e)

Q17. Box-and-whisker plots show the length (cm) of fish kept in two tanks.

Tank A: min 42, Q1 49, median 53, Q3 60, max 66.
Tank B: min 44, Q1 48, median 49, Q3 53, max 58.

There are 7 fish in Tank A with length greater than 60 cm.

(a) Find the total number of fish kept in Tank A. [1]
(b) Make one comment comparing the averages and one comment comparing the spread of the lengths in the two tanks. Use figures to support your answers. [4]
(c) A third tank Tank C has fish with the same median as Tank B but a smaller interquartile range than both Tank A and Tank B. Sketch a possible box plot for Tank C on the same scale. [2] [7 marks]

(a)

(b)

(c)

Q18. A box contains 6 white balls and 7 black balls, making 13 balls in total. Two balls are drawn from the box at random, one after the other, without replacement.

(a) Complete the probability tree diagram showing the two balls drawn and the corresponding probabilities. [2]
(b) Find the probability that the two balls drawn are of the same type. [2]
(c) Find the probability that at least one of the two balls drawn is white. [2]
(d) The two balls are returned to the box and a third ball is drawn at random. Given

that the first two balls drawn were both black, find the probability that the third ball drawn is white. [1] [7 marks]

(a)

(b)

(c)

(d)

Solutions

Q1. (a)

$(2x - 3)(x + 5) = 2x^2 + 10x - 3x - 15 = 2x^2 + 7x - 15$. [1]

$(x - 4)^2 = x^2 - 8x + 16$. [1]

Subtract and collect: $(2 - 1)x^2 + 15x - 31 = x^2 + 15x - 31$. [1]

Final answer: $x^2 + 15x - 31$

(b)

Group: $6xy - 9y = 3y(2x - 3)$; $4x - 6 = 2(2x - 3)$. [1]

Common binomial $(2x - 3)$: $(3y + 2)(2x - 3)$. [1]

Final answer: $(3y + 2)(2x - 3)$

(c)

$2x^2 - 1x - 3 = (2x + -3)(1x + 1) = 0$. [1]

$x = 3/2$ or $x = -1$. [1]

Final answer: $x = 3/2$ or $x = -1$

Q2. (a)

Numerator: $x^2 - 9 = (x - 3)(x + 3)$. [1]

Denominator: $3x^2 + 12x + 9 = (3x + 3)(x + 3)$. [1]

Cancel the shared $(x + 3)$ factor: $(x - 3) / (3x + 3)$. [1]

Final answer: $(x - 3) / (3x + 3)$

(b)

Cross-multiply: $y(x + 3) = 5x - 2$. [1]

Expand and group x: $yx + 3y = 5x - 2 \Rightarrow x(y - 5) = -2 - 3y \Rightarrow x(5 - y) = 2 + 3y$. [1]

Divide by $(5 - y)$: $x = (2 + 3y) / (5 - y)$. [1]

Final answer: $x = (2 + 3y) / (5 - y)$

Q3. (a)

$y = x^2 - 4x - 10$. [1]

Complete the square: $x^2 - 4x = (x - 2)^2 - 4$. [1]

$y = (x - 2)^2 - 4 - 10 = (x - 2)^2 - 14$. [1]

Final answer: $y = (x - 2)^2 - 14$

(b)

Turning point at $x = +2$, $y = -14$. [1]

Final answer: $(+2, -14)$

Q4. (a)

$y = (2)^3 - 4 \cdot (2) + 3 = 8 - 8 + 3 = 3$. [1]

Final answer: $y = 3$

(b)

Plot the 6 points from the table; join with a smooth curve. [1]

Apply scale: 2 cm per 1 unit on the x-axis; 1 cm per 2 units on the y-axis. [1]

Curve correctly shaped: cubic with one local maximum and one local minimum within the domain. [1]

Final answer: Smooth cubic curve plotted correctly

(c)

Read x-intercepts from the curve where $y = 0$; the cubic $x^3 - 4x + 3 = 0$ has up to 3 real roots. [1]

Roots: read off (accept ±0.1 each). (Exact values depend on the cubic; evaluate from the plotted curve.) [1]

Final answer: $x \approx$ values read from the graph

(d)

Draw a tangent at $x = 2$. Read rise/run from the tangent line. [1]

Analytic check: $dy/dx = 3x^2 - 4$; at $x = 2$ the gradient equals $3 \cdot 2^2 - 4 = 12 - 4 = 8$. Accept ±10%. [1]

Final answer: Gradient ≈ 8

(e)

To solve $x^3 - 4x + 3 = 3x - 1$ graphically, plot the line `y = 3x − 1` on the same grid. [1]

Line `y = 3x − 1` passes through $(0, -1)$ with gradient 3. [1]

Read x-coordinates of intersection points between the cubic curve and the line. (Accept ±0.1.) [1]

Final answer: $x \approx$ intersection x-values from the graph

Q5. (a)

Half of the x-coefficient: $(8)/2 = 4$. Square it: $4^2 = 16$. [1]

Adjust constant: $3 - 16 = -13$. Result: $x^2 + 8x + 3 = (x + 4)^2 + -13$. [1]

Final answer: $(x + 4)^2 + -13$

(b)

Vertex of $(x + 4)^2 + -13$ is at $(x, y) = (-4, -13)$ (minimum since leading coefficient is positive). [1]

Final answer: (-4, -13)

(c)

Line of symmetry passes through the vertex; vertical line $x = -4$. [1]

Final answer: $x = -4$

Q6. (a)

Gradient of AB = $(y_2 - y_1) / (x_2 - x_1) = (5 - 1)/(1 - -3) = 4/4 = 1$. [1]

Use point A(-3, 1): $y - 1 = 1(x - -3)$. Rearrange: $y = 1x + 4$. [1]

Final answer: $y = 1x + 4$

(b)

Parallel line has the same gradient 1. Use point C(5, 8): $y - 8 = 1(x - 5)$. [1]

Rearrange: $y = 1x + 3$. [1]

Final answer: $y = 1x + 3$

(c)

Set parallel line equal to second line: $1x + 3 = -1x + -5$. [1]

Solve: $(1 - -1)x = -5 - 3 \rightarrow x = -4$; substitute back: $y = 1 \cdot -4 + 3 = -1$. [1]

Final answer: (-4, -1)

(d)

Distance AB = $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} = \sqrt{(16 + 16)} = \sqrt{32}$. [1]

$AB^2 = 32$; $AB = \sqrt{32}$. [1]

Final answer: $AB = \sqrt{32}$

Q7. (a)

Angle ABC and angle ADC are opposite angles of cyclic quadrilateral ABCD. [1]

Angle ABC = 180° − 118° = 62° (opposite angles of cyclic quadrilateral are supplementary). [1]

Final answer: Angle ABC = 62°

(b)

By the alternate segment theorem, the angle between tangent TA and chord AB equals the angle in the alternate segment subtending AB. [1]

Angle ACB = angle BAT = 40° (alternate segment theorem). [1]

Final answer: Angle ACB = 40°

(c)

Non-reflex angle AOC = 2 × angle ABC = 2 × 62° = 124° (angle at centre = 2 × angle at circumference). [1]

Reflex angle AOC = 360° − 124° = 236°. [1]

Final answer: Reflex angle AOC = 236°

Q8. (a)

Angle ACB = angle DAB (given), and angle ABC = angle DBA (common angle). [1]

∴ △ABC ~ △DBA (AA). [1]

Final answer: △ABC ~ △DBA (AA)

(b)

From △ABC ~ △DBA: AB / DB = BC / AB, so DB = AB^2 / BC = 6^2 / 2 = 18. [1]

$CD = DB - BC = 18 - 2 = 16$ cm. [1]
Final answer: $CD = 16$ cm

(c)
Linear ratio $\triangle ABC : \triangle DBA = BC : AB = 2 : 6$; area ratio $= (2 / 6)^2$. [1]
Area $\triangle ABC = 36 \times 2^2 / 6^2 = 4$ cm^2. [1]
Final answer: Area $\triangle ABC = 4$ cm^2

Q9. (a)
$OP = OQ$ (radii of the larger circle); $OS = OR$ (radii of the smaller circle); angle OSP = angle $ORQ = 90°$ (tangent perpendicular to radius at point of contact). [1]
$\therefore \triangle PSO \cong \triangle QRO$ (RHS). [1]
Final answer: $\triangle PSO \cong \triangle QRO$ (RHS)

(b)
In $\triangle QRO$ with right angle at R: $OQ = 17$ cm, $OR = 8$ cm. [1]
By Pythagoras, $RQ^2 = OQ^2 - OR^2 = 17^2 - 8^2 = 225$, so $RQ = 15$ cm. [1]
Area $\triangle QRO = ½ \times OR \times RQ = ½ \times 8 \times 15 = 60$ cm^2. [1]
Final answer: Area $\triangle QRO = 60$ cm^2

Q10. (a)
Cosine rule: $AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle ABC) = (20)^2 + (30)^2 - 2(20)(30) \cdot \cos(95°)$. [1]
$AC^2 \approx 1404.6$; $AC \approx 37.5$ m. [1]
Final answer: $AC \approx 37.5$ m

(b)
Sine rule: $\sin(\angle BAC) / BC = \sin(\angle ABC) / AC$; $\sin(\angle BAC) = (30) \cdot \sin(95°) / 37.5$. [1]
$\angle BAC \approx 52.9°$. [1]
Final answer: $\angle BAC \approx 52.9°$

(c)
Area $\triangle ABC = (1/2) \cdot AB \cdot BC \cdot \sin(\angle ABC) = (1/2)(20)(30) \cdot \sin(95°) \approx 299$ m^2. [1]
Shortest distance from B to AC: $h = 2 \cdot \text{Area} / AC = 2 \cdot 299 / 37.5 \approx 15.9$ m. [1]
Final answer: ≈ 15.9 m

(d)
Greatest elevation occurs at the foot of the perpendicular from A to BC. Let that distance be d;
$d = 2 \cdot \text{Area}(\triangle ABC) / BC = 2 \cdot 299 / 30 \approx 19.9$ m. [1]
$\tan \vartheta = \text{pole} / d = (12) / 19.9$. [1]
$\vartheta \approx 31.1°$. [1]
Final answer: $\approx 31.1°$

Q11. (a)

Q is due east of P, so PQ has bearing 90° from P. Angle QPR = (45° − 90°) reflected to interior = (90 − 45)° but the bearing of R from P is 45°, north of east, so ∠QPR = 90 − 45 taken as |...| = 45°. [1]

From Q, PQ has bearing 270° (P is due west of Q). ∠PQR = 315° − 270° = 45°. ∠PRQ = 180° − 45° − 45° = 90°. [1]

Final answer: ∠QPR = 45°, ∠PQR = 45°, ∠PRQ = 90°

(b)

Sine rule: QR / sin(∠QPR) = PQ / sin(∠PRQ); QR = (60) · sin(45°) / sin(90°). [2]

QR ≈ 42.4 m. [1]

Final answer: QR ≈ 42.4 m

(c)

Area △PQR = (1/2) · PQ · QR · sin(∠PQR) = (1/2)(60)(42.4) · sin(45°) ≈ 900 m^2. [1]

Shortest distance from R to PQ: h = 2 · Area / PQ = 2 · 900 / (60) ≈ 30 m. [1]

Final answer: Area ≈ 900 m^2, h ≈ 30 m

Q12. (a)

Arc length s = r · ϑ = (15)·(1.2) = 18 cm. [1]

Final answer: arc AB = 18 cm

(b)

Sector area = (1/2) · r^2 · ϑ = (1/2)·$(15)^2$·(1.2). [1]

= 135 cm^2. [1]

Final answer: ≈ 135 cm^2

(c)

M is midpoint of OB, so OM = OB/2 = 7.5 cm. Cosine rule in △OAM with ∠AOM = (1.2) rad:

$AM^2 = OA^2 + OM^2 - 2 \cdot OA \cdot OM \cdot \cos(\vartheta) = (15)^2 + (7.5)^2 - 2(15)(7.5)\cdot\cos(1.2)$. [1]

AM ≈ 14.1 cm. [1]

Final answer: AM ≈ 14.1 cm

(d)

Triangle OAM area = (1/2) · OA · OM · sin(ϑ) = (1/2)(15)(7.5)·sin(1.2) ≈ 52.4 cm^2. [1]

Smaller region = sector − triangle ≈ 135 − 52.4. [1]

≈ 82.6 cm^2. [1]

Final answer: ≈ 82.6 cm^2

Q13. (a)

MYR received from initial exchange: 2500 × 3.00 = 7500 MYR. [1]

Total spent on accommodation, food and fuel: 3000 + 100 × 2.40 = 3000 + 240 = 3240 MYR.

MYR remaining = 7500 − 3240 = 4260 MYR. [1]

Final answer: 4260 MYR

(b)
He receives back SGD 80 which is $80 \times 3.10 = 248$ MYR converted. That leaves $4260 - 248 = 4012$ MYR used on other items. [1]
Return trip needs $100 \times 2.40 = 240$ MYR for fuel. Since $4260 \geq 240$, yes he has enough. [1]
Final answer: Yes, sufficient MYR remaining

(c)
Bank effective rate per SGD $= 2.95 \times (1 - 1.5/100) = 2.95 \times 0.985$. [1]
$= 2.906$ MYR per SGD. [1]
Since $2.906 < 3.00$, the original rate of 3.00 gives more MYR per SGD; original rate is better. [1]
Final answer: Original rate (3.00 MYR/SGD) is better than bank effective rate (2.906 MYR/SGD)

Q14. (a)
Marked price = cost $\times (1 + 50/100) = 800 \times 1.5 = \$1200$. [1]
Final answer: \$1200

(b)
Selling price = marked $\times (1 - 30/100) = 1200 \times 0.7 = \$840$. [1]
Final answer: \$840

(c)
Selling price equals cost $\times (1 + 5/100) = 800 \times 1.05 = \$840$. Working backwards: cost = sold $\div (1 + 5/100)$. [1]
Cost $= \$840 / 1.05 = \$800$. [1]
Final answer: \$800

Q15. (a)
The compound interest formula is $A = P(1 + r/100)^n$ where P is the principal, r is the interest rate per period (in %), and n is the number of periods. [1]
Final answer: $A = P(1 + r/100)^n$

(b)
Substitute P = \$10000, r = 4, n = 5: $A = 10000 \times (1 + 4/100)^5$. [1]
$A = 10000 \times (1.04)^5$. [1]
$A = \$12166.53$. [1]
Final answer: \$12166.53

Q16. (a)
Running totals (cumulative frequency): (10, 20), (20, 39), (30, 52), (40, 67), (50, 82), (60, 100). [1]
Final answer: cumulative frequencies: [20, 39, 52, 67, 82, 100]

(b)

Plot points (upper class boundary, cumulative frequency): (10, 20), (20, 39), (30, 52), (40, 67), (50, 82), (60, 100), anchored at (0, 0). [1]

Join the points with a smooth S-shaped curve. [1]

Final answer: Smooth cumulative-frequency curve drawn

(c)

Median: read x at cumulative frequency = n/2 = 50. [1]

Q1 at cf = n/4 = 25; Q3 at cf = 3n/4 = 75; IQR = Q3 - Q1. [1]

20th percentile: read x at cf = 20/100 × n = 20. [1]

State each read-off value with its unit. [1]

Final answer: Median, IQR, and percentile read from curve

(d)

Read cf at the threshold value 30; count above threshold = 100 − (cf at threshold). [1]

State the numeric count from the curve. [1]

Final answer: n − (cf at threshold)

(e)

Same median means the curve passes through the same Q2 point. [1]

Larger IQR means the curve is less steep / flatter between Q1 and Q3. [1]

Final answer: Same median point; flatter shoulders in the Q1 to Q3 region

Q17. (a)

Threshold 60 equals A's Q3, so a quarter (1/4) of A's items lie above it. With 7 items above, total = 4 × 7 = 28. [1]

Final answer: 28

(b)

Centre: median A = 53, median B = 49. A is higher by 4. [1]

Centre comment with context: A items take, on average, a higher value than B items. [1]

Spread: IQR(A) = 60 − 49 = 11; IQR(B) = 53 − 48 = 5. A has the larger IQR by 6. [1]

Spread comment with context: A items are more varied / spread out than B items. [1]

Final answer: A has higher median (53 vs 49) and larger IQR (11 vs 5)

(c)

C has the same median as B = 49; smaller IQR than both A (11) and B (5). [1]

Sketch a box centred at 49 with a narrower box (e.g. width 2 or 3) than B's width of 5; whiskers within reasonable data range. [1]

Final answer: box plot for C: median 49, IQR < 5

Q18. (a)

First draw: P(A)=6/13, P(B)=7/13. [1]

Second draw without replacement: after A → P(A)=5/12, P(B)=7/12; after B → P(A)=1/2,

P(B)=1/2. [1]
Final answer: Tree completed

(b)
P(both A) = $6/13 \times 5/12 = 5/26$; P(both B) = $7/13 \times 1/2 = 7/26$. [1]
P(same) = $5/26 + 7/26 = 6/13$. [1]
Final answer: 6/13

(c)
P(no A) = P(both B) = $7/13 \times 1/2 = 7/26$. [1]
P(at least one A) = $1 - 7/26 = 19/26$. [1]
Final answer: 19/26

(d)
After returning, composition is restored: 6 A and 7 B in 13. P(third = A) = $6/13 = 6/13$. [1]
Final answer: 6/13